Pinger Bearing

This page describes the math used to calculate our bearing towards the pinger using our hydrophones. A solution is derived in 2 dimensions to show how it works, then our final result is extrapolated to 3 dimensions.

Assumptions

Planar Wave

Imagine dropping a rock in the middle of a pond. At first the ripples look very circular, but by the time they reach the shore they look like straight lines. We assume that the wave front of the ping is planar, which basically means that even though the wave front is curved, the curvature is small enough from the perspective of the hydrophones such that the wave front actually looks flat. This makes our calculations a lot easier.

3D Result

Extrapolating the 2D result into 3 dimensions, we have:

H_0 is at location $(0,0,0)$
H_x is at location $(x,0,0)$
H_y is at location $(0,y,0)$
H_z is at location $(0,0,z)$

Given that our hydrophones receive the ping at times t_0, t_x, t_y, andt_z respectively, we can calculate:

$$ \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} = c_s \begin{bmatrix} t_0 - t_x \\ t_0 - t_y \\ t_0 - t_y \end{bmatrix} $$

$$ \begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} = \begin{bmatrix} d_x/x \\ d_y/y \\ d_z/z \end{bmatrix} $$

2D-Derivation

The speed of sound of water is defined as c_s. We have 3 hydrophones, located at the following positions:

H_0 is at location $(0,0)$
H_x is at location $(x,0)$
H_y is at location $(0,y)$

We shall simulate a wave traveling towards our hydrophone array. First, it hits H_x at time t_x

The wave continues, hitting H_y at time t_y

Finally, the wave hits H_0 at time t_0

Because the wave emanating from the pinger hit H_x before it hit H_0, we know that H_x must be closer to the pinger than H_0. In fact, H_x must be exactly $d_x = c_s(t_0 - t_x)$ meters closer to the pinger than H_0. Likewise, H_y is $d_y = c_s(t_0 - t_y)$ meters closer to the pinger than H_0.

Our main goal is to calculate a vector pointing from the H_0 to the pinger. This vector is perpendicular to the wave front, and is shown in blue in the diagram below:

We can also describe the vector in i,j notation: $$ \begin{bmatrix} \hat{i} \\ \hat{j} \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} $$

Luckily, it is pretty simple to calculate $\hat{i}$ and $\hat{j}$: $$ \begin{bmatrix} \hat{i} \\ \hat{j} \end{bmatrix} = \begin{bmatrix} d_x/x \\ d_y/y \end{bmatrix} $$