

# Trilateration

Below is the math for calculating the location of the pinger in the water relative to our submarine. Because we know the absolute location of the pinger in the pool, we can calculate the sub's position in the pool.

[Original derivation](#) by Brian Moore.

## Problem Setup

The derivation is based on the assumption that we have 4 hydrophones. One is considered the reference hydrophone, while the others are located along the x, y, and z axes.

$h_0$  is at location  $(0,0,0)$

$h_x$  is at location  $(\delta,0,0)$

$h_y$  is at location  $(0,\epsilon,0)$

$h_z$  is at location  $(0,0,\zeta)$

We define the ping source is at location  $\mathbf{l}_{\text{pinger}} = (x,y,z)$

When a ping is received by the hydrophones, the hardware outputs delta-timestamps  $\Delta t_x$ ,  $\Delta t_y$ ,  $\Delta t_z$ , which corresponds to the difference in time between the ping was received by  $h_0$  and  $h_{\{x,y,z\}}$ , respectively.

Let's define  $p_0$  as the absolute distance between  $h_0$  and the pinger at location  $(x,y,z)$ .  

$$p_0 = \sqrt{x^2 + y^2 + z^2}$$

These three time differences are multiplied by the speed of sound in water ( $c_s$ ) to determine the difference in distance between the the reference hydrophone and pinger and each other hydrophone  $h_{\{x,y,z\}}$ .  

$$\Delta x = \Delta t_x * c_s \quad \Delta y = \Delta t_y * c_s \quad \Delta z = \Delta t_z * c_s$$
 In other words,  $h_x$  is  $\Delta x$  meters farther from the pinger than  $h_0$ , and  $h_0$  is  $p_0$  meters from the pinger.

The final calculations for  $x$ ,  $y$ , and  $z$  will be in terms of  $\Delta x$ ,  $\Delta y$ , and  $\Delta z$

Let's define the distances from the other hydrophones to the pinger:  

$$p_x = \sqrt{(x-\delta)^2 + y^2 + z^2} \quad p_y = \sqrt{x^2 + (y-\epsilon)^2 + z^2} \quad p_z = \sqrt{x^2 + y^2 + (z-\zeta)^2}$$

Let's put this information together:  

$$\Delta x = p_0 - p_x = \sqrt{x^2 + y^2 + z^2} - \sqrt{(x-\delta)^2 + y^2 + z^2}$$

$$\Delta y = p_0 - p_y = \sqrt{x^2 + y^2 + z^2} - \sqrt{x^2 + (y-\epsilon)^2 + z^2}$$

$$\Delta z = p_0 - p_z = \sqrt{x^2 + y^2 + z^2} - \sqrt{x^2 + y^2 + (z-\zeta)^2}$$

Since we know  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ ,  $\delta$ ,  $\epsilon$ , and  $\zeta$  we can solve for  $x$ ,  $y$ ,  $z$ !

## Solving for Position

Using  $\Delta x$  and its measurements, we will derive a formula for  $x$ . The same steps can be used for solving  $y$  and  $z$ .

First, let's start with the final equation from the previous section:  $\Delta x = \sqrt{x^2 + y^2 + z^2} - \sqrt{(x-\Delta)^2 + y^2 + z^2}$  Also recall that  $p_0 = \sqrt{x^2 + y^2 + z^2}$   $p_x = \sqrt{(x-\Delta)^2 + y^2 + z^2}$  Therefore  $p_0^2 - p_x^2 = (x^2 + y^2 + z^2) - ((x-\Delta)^2 + y^2 + z^2)$  Simplifying, we get  $p_0^2 - p_x^2 = x^2 - (x-\Delta)^2 = x^2 - (x^2 - 2\Delta x + \Delta^2) = 2\Delta x - \Delta^2$  Note that  $p_0^2 - p_x^2 = (p_0 + p_x)(p_0 - p_x)$   $p_0 - p_x = \frac{p_0^2 - p_x^2}{p_0 + p_x}$  Therefore  $p_0 - p_x = \frac{2\Delta x - \Delta^2}{p_0 + p_x}$

Recall that  $\Delta x = p_0 - p_x$   $p_x = p_0 - \Delta x$

Combining this with the previous equation, we can substitute  $\Delta x$  for  $p_0 - p_x$  on the left side and  $p_0 - \Delta x$  for  $p_x$  on the right side, which gives us  $\Delta x = \frac{2\Delta x - \Delta^2}{2p_0 - \Delta x}$  Let's now take this result and solve for  $x$   $\Delta x = \frac{2\Delta x - \Delta^2}{2p_0 - \Delta x}$   $\Delta x (2p_0 - \Delta x) = 2\Delta x - \Delta^2$   $2p_0 \Delta x - \Delta x^2 = 2\Delta x - \Delta^2$   $\Delta x (2p_0 - \Delta x) + \Delta^2 = 2\Delta x$   $\Delta x = \frac{2\Delta x}{2p_0 - \Delta x + \Delta^2}$  Following this same technique, we can also derive equations for  $y$  and  $z$ .  $x = \frac{\Delta x (2p_0 - \Delta x) + \Delta^2}{2\Delta}$   $y = \frac{\Delta y (2p_0 - \Delta y) + \epsilon^2}{2\epsilon}$   $z = \frac{\Delta z (2p_0 - \Delta z) + \zeta^2}{2\zeta}$  It can be seen that  $x$ ,  $y$ , and  $z$  are dependent not only in the time delays, but also on  $p_0$ . Therefore, it is necessary to find an expression for  $p_0$  in terms of  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ ,  $\Delta$ ,  $\epsilon$ , and  $\zeta$ .

First, remember that  $p_0 = \sqrt{x^2 + y^2 + z^2}$   $p_0^2 = x^2 + y^2 + z^2$  We can solve for  $p_0^2$  using  $x^2$ ,  $y^2$ , and  $z^2$ . Let's take our result for  $x$ , calculate  $x^2$ , and expand.  $x = \frac{\Delta x (2p_0 - \Delta x) + \Delta^2}{2\Delta}$   $x^2 = \left( \frac{\Delta x (2p_0 - \Delta x) + \Delta^2}{2\Delta} \right)^2$   $x^2 = \frac{(\Delta x (2p_0 - \Delta x) + \Delta^2)^2}{4\Delta^2}$   $x^2 = \frac{(\Delta x^2 (2p_0 - \Delta x)^2 + 2\Delta x \Delta^2 (2p_0 - \Delta x) + \Delta^4)}{4\Delta^2}$   $x^2 = \frac{4p_0^2 \Delta x^2 - 4p_0 \Delta x^3 + 4p_0 \Delta^2 \Delta x + \Delta x^4 - 2\Delta^2 \Delta x^2 + \Delta^4}{4\Delta^2}$   $x^2 = \frac{p_0^2 (4\Delta x^2) + p_0 (-4\Delta x^3 + 4\Delta^2 \Delta x) + (\Delta x^2 - \Delta^2)^2}{4\Delta^2}$   $x^2 = p_0^2 \frac{4\Delta x^2}{4\Delta^2} + p_0 \frac{-4\Delta x^3 + 4\Delta^2 \Delta x}{4\Delta^2} + \frac{(\Delta x^2 - \Delta^2)^2}{4\Delta^2}$   $x^2 = p_0^2 \frac{\Delta x^2}{\Delta^2} + p_0 \frac{\Delta x}{\Delta} \frac{(\Delta x^2 - \Delta^2)}{\Delta^2} + \frac{(\Delta x^2 - \Delta^2)^2}{4\Delta^2}$

We can derive a similar equation for  $y^2$  and  $z^2$   $x^2 = p_0^2 \frac{\Delta x^2}{\Delta^2} + p_0 \frac{\Delta x}{\Delta} \frac{(\Delta x^2 - \Delta^2)}{\Delta^2} + \frac{(\Delta x^2 - \Delta^2)^2}{4\Delta^2}$   $y^2 = p_0^2 \frac{\Delta y^2}{\epsilon^2} + p_0 \frac{\Delta y}{\epsilon} \frac{(\Delta y^2 - \epsilon^2)}{\epsilon^2} + \frac{(\Delta y^2 - \epsilon^2)^2}{4\epsilon^2}$   $z^2 = p_0^2 \frac{\Delta z^2}{\zeta^2} + p_0 \frac{\Delta z}{\zeta} \frac{(\Delta z^2 - \zeta^2)}{\zeta^2} + \frac{(\Delta z^2 - \zeta^2)^2}{4\zeta^2}$

From:

<https://robosub.eecs.wsu.edu/wiki/> - **Palouse RoboSub Technical Documentation**

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